Product differentiation

Industrial Organization

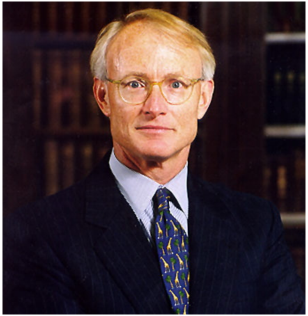
Frieder Neunhoeffer

ISEG, 2025/26

Introduction

Michael Porter (Competitive advantage, 1985)

Competitive advantage stems from the many discrete activities a firm performs in designing, producing, marketing, delivering and supporting its product. Each of these activities can contribute to a firm's relative cost position and create a basis for differentiation.



A competitive advantage should be

- Significant and rare
- Defendable

Introduction

Three forms of competitive advantage:

- Differentiation
- Costs
- A combination of the two

Two ways to gain a competitive advantage through differentiation:

- By creating a real difference between products
- By influencing consumer preferences (e.g., advertising)

Various forms of differentiation

"Horizontal" differentiation:

- Consumers have different tastes (e.g., color)
- If the products are all sold at the same price, consumers may choose differently

"Vertical" differentiation:

- Different qualities
- Consumers agree on the ranking of the goods in terms of quality
- If all sold at the same price, consumers choose the product with the "highest" quality

Lancaster's approach (1966):

- A good is a bundle of characteristics

Sources of differentiation

Different possibilities of differentiation:

- the product (shape, style, design, reliability, etc.)
- the service (ordering, delivery, installation, etc.)
- the staff, the point of sale, the brand image, ...

Notion of "positioning" in marketing:

Brand strategy $\rightarrow$ the consumer must be able to identify the characteristics of the product in relation with their needs.

Differentiation strategies

We are going to examine the following questions:

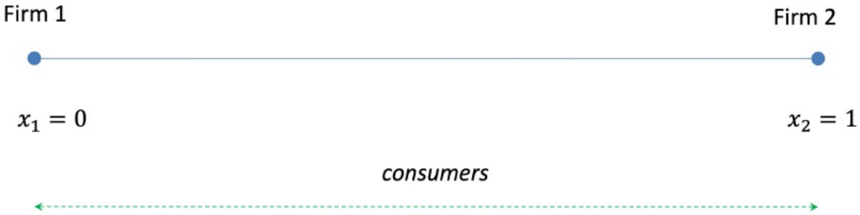
- How does differentiation affect competition among firms?
- When differentiation is *endogenous*, what is the equilibrium? Do firms actually choose to differentiate themselves?
- Are the products offered in equilibrium close or far apart?
- What is the effect of differentiation on entry of new players?

The Hotelling model (1929)

- A “street” or a “space of tastes” represented by the interval $[0, 1]$
- A mass 1 of consumers are distributed uniformly along this interval
- Two firms, 1 and 2, sell a good (the same good) on this street
- Firms compete in prices
- Same marginal cost of production c
- Consumers buy 0 or 1 unit of the good
- Utility of the good for a consumer: v
- $\rightarrow$ But there are (e.g., quadratic) transportation costs t consumers pay to get the good
- So, net utility $= v - p - \textit{transportation cost}$

Exogenous location of firms

Firm 1 is at $x_1 = 0$ and firm 2 at $x_2 = 1$.

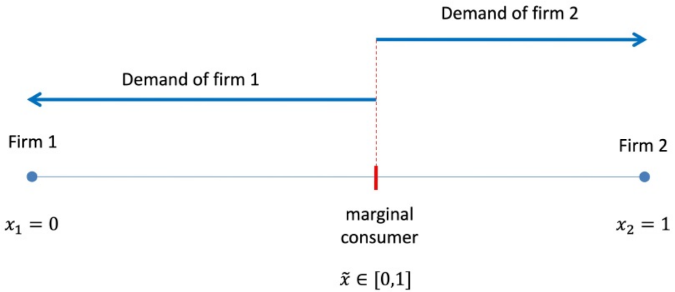


Method for calculating demand: the marginal consumer

Definition

In the presence of consumers with heterogeneous tastes, the *marginal consumer* is the consumer who is indifferent between two possible choices.

Where is the marginal consumer approximately located?



Demand functions

- The marginal consumer $\tilde{x}$ is defined by

$$v - \left(p_1 + (\tilde{x} - 0)^2 t\right) = v - \left(p_2 + (1 - \tilde{x})^2 t\right)$$

so,

$$\tilde{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

- From this, we can derive the demand of firm 1 (if prices are not too different)

$$D_1(p_1, p_2) = \tilde{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

- The demand of firm 2 is $D_2 = 1 - \tilde{x} = 1 - D_1$

Profits and reaction functions

We start by defining the profit functions:

$$\pi_i = (p_i - c)\left(\frac{1}{2} + \frac{p_j - p_i}{2t}\right)$$

- Firm i maximizes its profit π_i, taking the rival's price p_j as given
- The first order condition gives the optimal price for firm i as a function of rival's price p_j

Reaction function of firm i (best response):

$$p_i = R_i(p_j) = \frac{c + t + p_j}{2}$$

Nash equilibrium

The Nash equilibrium corresponds to the intersection of the reaction functions.

We have

$$p_i = R_i(R_j(p_i)) \quad \text{with} \quad p_i = R_i(p_j) = \frac{c + t + p_j}{2}$$

What is the equilibrium price?

$$p^\star = c + t$$

$\rightarrow$ The equilibrium price increases with t

Conclusion

When firms' locations are fixed, an increase in the level of differentiation (measured by t) increases firms' market power.

When firms choose where to locate

$\rightarrow$ Location decisions are endogenous ($\neq$ exogenous)

We study a **two-stage game**:

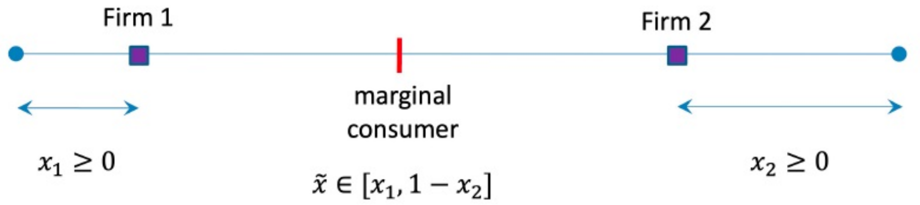
1. Firms choose locations
2. Given locations, firms set prices

We look for the subgame perfect equilibrium using backward induction:

- Solve price competition (last stage)
- Then, solve location choices (first stage)
- Firms anticipate second-stage equilibrium (*subgame perfection*)

Endogeneous locations

Firm 1 is located at x_1 of the left end and firm 2 at x_2 of the right end.



Stage 2: choice of prices

First determine each firm's demand for given prices.

- The marginal consumer $\tilde{x}$ is given by:

$$v - \left(p_1 + (\tilde{x} - x_1)^2 t\right) = v - \left(p_2 + (1 - x_2 - \tilde{x})^2 t\right)$$

so, the marginal consumer is located at

$$\tilde{x} = x_1 + \frac{1 - x_1 - x_2}{2} + \frac{p_2 - p_1}{2t(1 - x_1 - x_2)}$$

- If $0 \leq \tilde{x} \leq 1$, the demands for firm 1 and firm 2 are

$$D_1 = \tilde{x} \quad \text{and} \quad D_2 = 1 - \tilde{x}$$

Stage 2: choice of prices

Next, we determine the reaction functions.

- The profit function of firm i ($i = 1, 2$) is:

$$\pi_i = (p_i - c)\left(x_i + \frac{1 - x_1 - x_2}{2} + \frac{p_j - p_i}{2t(1 - x_1 - x_2)}\right)$$

- Firm i maximizes its profit with respect to p_i, taking its rival's price p_j as given
- We find the two FOCs for firm 1 and firm 2, which gives two reaction functions

Stage 2: choice of prices

The **intersection of the reaction functions** gives the equilibrium prices at stage 2:

$$p_1^\star = c + t(1 - x_1 - x_2)\left(1 + \frac{x_1 - x_2}{3}\right)$$

$$p_2^\star = c + t(1 - x_1 - x_2)\left(1 + \frac{x_2 - x_1}{3}\right)$$

Stage 1: choice of location

- At stage 1, firm 1 chooses her location taking the location of firm 2 as given
- She anticipates the equilibrium price of stage 2
- Her profit function is

$$\pi_1\left(x_1, p_1(x_1, x_2), p_2(x_1, x_2)\right) = (p_1^\star - c)D_1$$

- Therefore, her profit maximization problem with respect to x_1 is:

$$\max_{x_1} \left(p_1^\star(x_1, x_2) - c\right) D_1\left(x_1, x_2, p_1^\star(x_1, x_2), p_2^\star(x_1, x_2)\right)$$

- Indeed, the location choice affects profit in two different ways:
  - Direct effect: π_1 depends on x_1
  - Indirect effects: π_1 depends on p_1 and p_2, which in turn depend on x_1

Stage 1 (choice of location): Derivative of profit

$$\pi_1 = (p_1^\star - c)D_1 = \big(p_1^\star(x_1) - c\big)D_1\big(x_1, p_1^\star(x_1), p_2^\star(x_1)\big)$$

Both terms depend on $x_1 \Rightarrow$ product rule:

$$\frac{d\pi_1}{dx_1} = \underbrace{\frac{\partial p_1^\star}{\partial x_1}D_1}_{(A)} + \underbrace{(p_1^\star - c)\frac{dD_1}{dx_1}}_{(B)}$$

Demand depends on x_1, $p_1^\star$, $p_2^\star$:

$$\frac{dD_1}{dx_1} = \frac{\partial D_1}{\partial x_1} + \frac{\partial D_1}{\partial p_1}\frac{\partial p_1^\star}{\partial x_1} + \frac{\partial D_1}{\partial p_2}\frac{\partial p_2^\star}{\partial x_1}$$

$$\rightarrow \frac{d\pi_1}{dx_1} = \frac{\partial p_1^\star}{\partial x_1}D_1 + (p_1^\star - c)\left(\frac{\partial D_1}{\partial x_1} + \frac{\partial D_1}{\partial p_1}\frac{\partial p_1^\star}{\partial x_1} + \frac{\partial D_1}{\partial p_2}\frac{\partial p_2^\star}{\partial x_1}\right)$$

Stage 1 (choice of location): Envelope theorem simplifies the derivative

$$\frac{d\pi_1}{dx_1} = (p_1^\star - c)\left(\frac{\partial D_1}{\partial x_1} + \frac{\partial D_1}{\partial p_2}\frac{\partial p_2^\star}{\partial x_1}\right) + \frac{\partial p_1^\star}{\partial x_1}\underbrace{\left[D_1 + (p_1^\star - c)\frac{\partial D_1}{\partial p_1}\right]}_{=\frac{\partial \pi_1}{\partial p_1}}$$

Stage 2 optimal-pricing implies (envelope theorem):

$$\left.\frac{\partial \pi_1}{\partial p_1}\right|_{p_1 = p_1^\star} = 0$$

Therefore,

$$\frac{d\pi_1}{dx_1} = (p_1^\star - c)\left(\underbrace{\frac{\partial D_1}{\partial x_1}}_{\text{direct effect } (+)} + \underbrace{\frac{\partial D_1}{\partial p_2}\frac{\partial p_2^\star}{\partial x_1}}_{\text{indirect effect } (-)}\right)$$

Stage 1: choice of location

In a game with **several stages**, we potentially have:

- Direct effects: when variables chosen in the first stage directly affect the profit functions
- Indirect effects (strategic effects): when variables chosen in the first stage affect the strategies chosen in later stages, which in turn affect profits

Here:

- Direct effect $(+)$ is a *demand effect*
- Indirect or strategic effect $(-)$ is the *intensification of competition*

Stage 1: choice of location

Plugging in the EQ prices and D_1, we find that the strategic effect $(-)$ always dominates the direct effect of market share $(+)$.

What are the firms' differentiation strategies in equilibrium?

$\rightarrow$ Firms choose maximum differentiation

In a context of price competition, firms have strong incentives to differentiate themselves in order to soften competition.

Although they also have an incentive to "imitate" their rivals in order to capture market share.

How do firms' location choices compare to the social optimum?

The socially optimal locations are those that minimize production and transportation costs. These costs are minimized when the two firms are located at $1/4$ and at $3/4$.

Do firms differentiate themselves *too much* or *too little*?

There is too much differentiation.

If prices are exogenous...

Assume that prices are exogenous (fixed) and the same for both firms.

What are the firms' locations in equilibrium?

In EQ, the two firms produce the same level of variety $\rightarrow$ there is minimal differentiation (classic Hotelling ice-cream on the beach example)

The example of television

If we apply these results to competition between TV channels on the audience market...

- Where do you expect more differentiation? Channels financed (purely) by subscription fees or channels financed (purely) by advertising
- Differentiation is minimal for TV channels financed by advertising (think of "fixed pricing")

Other dimensions of competition that could change this result:

- Differentiation to reduce competition in the quality of TV programs
- Differentiation to prevent viewers from switching to the channel with the least advertising

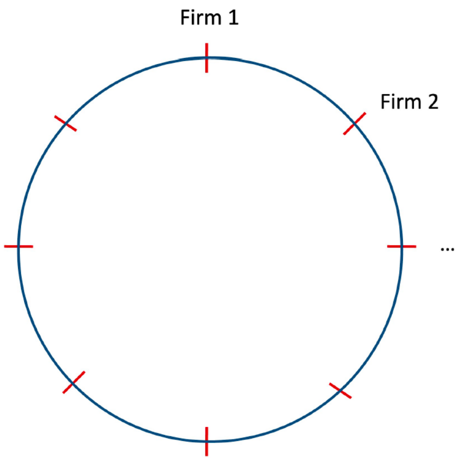
Salop model (1979)

Salop model (1979) of a circular city.

- A differentiation circle with perimeter equal to 1 (a "circular city")
- A mass 1 of consumers are uniformly distributed around the circle
- n identical firms are also located on the circle $\rightarrow$ we assume that they are uniformly distributed around the circle
- Firms compete in prices
- Marginal cost c
- We assume a free entry condition and denote by f the fixed cost of entry

How many firms enter in equilibrium? Are there too few or too many entries (e.g., if there were only private hospitals)?

The circular city



Demand functions

- Consider the pricing decision for firm i
- Firm i has two close "rivals", which propose the same price p (symmetry assumption)
- The two rivals are both located at a distance of $1/n$ from firm i
- We start by determining the location $\tilde{x}$ of the indifferent consumer between firm i and its rival located $1/n$ further away:

$$p_1 + t\tilde{x} = p + t\left(\frac{1}{n} - \tilde{x}\right)$$

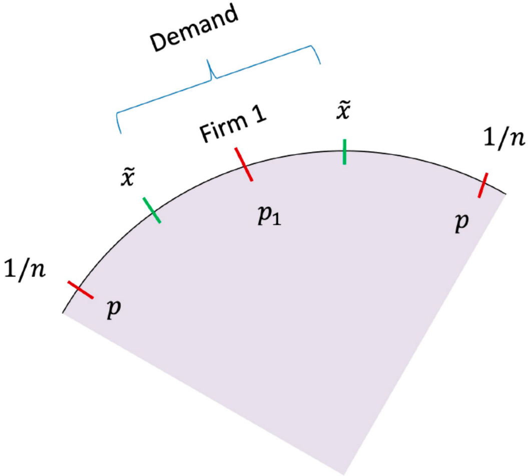
- That gives

$$\tilde{x} = \frac{1}{2n} + \frac{p - p_1}{2t}$$

- So, D_1 is

$$D_1(p_1, p) = 2\tilde{x}$$

The circular city



Price equilibrium

- Firm 1's profit is:

$$\pi_1 = (p_1 - c)\left(\frac{1}{n} + \frac{p - p_1}{t}\right)$$

- We solve for firm 1's best response function:

$$p_1^\star(p) = \frac{c + p + \frac{t}{n}}{2}$$

- We assume a symmetric equilibrium where $p_1 = p$, so

$$p^\star = c + \frac{t}{n}$$

- The equilibrium profit is (with a fixed cost f):

$$\pi^\star(n) = \frac{t}{n^2} - f$$

Equilibrium with free entry

How can we find the number of entrants in equilibrium?
$\rightarrow$ There is entry as long as the profit of a new entrant is strictly positive

We find the number of firms n that satisfies the zero-profit condition in order to obtain the number of entrants in equilibrium:

$$n^\star = \sqrt{\frac{t}{f}}$$

So, the long-run equilibrium price is

$$p^\star = c + \sqrt{tf}$$

What's the effect of f? What's the effect of t?

Comparison with social optimum

We will prove in problem set 4, that from the point of view of social welfare, there is too much entry!

How can we explain this result?

- Private and social incentives are not aligned
- New entrants offer new varieties but also steal customers from rivals (business stealing)

Brand proliferation

Consider the **circular city model** again.

Could firms decide to increase the number of products to prevent competitors from entering the market (brand proliferation strategy)?

For instance, in 1972, the top six companies in the US breakfast cereal market held 95% of the market. Between 1950 and 1972, they launched more than 80 different brands.

The FTC charged the 4 companies with abuse of a dominant position (but lost the case).

Vertical differentiation

- In the **horizontal differentiation** model, firms produce different types of products but offer the same quality.
- **Vertical differentiation** necessarily implies asymmetries: there are high quality providers and low quality providers.
- Is the principle of maximum differentiation still valid?

Model of vertical differentiation

- Two firms, 1 and 2, produce goods of different qualities: s_1 (firm 1) and s_2 (firm 2)
- Marginal cost of production c!
- Production cost of quality is 0
- Firms first choose their product quality, then simultaneously choose their prices (two-stage game)
- All consumers value quality, but at different levels $\rightarrow$ consumers' valuation for quality uniformly distributed on $[\underline{\theta}, \bar{\theta}]$, with

$$\underline{\theta} \geq 0 \quad \text{and} \quad \bar{\theta} = \theta + 1$$

- A consumer with valuation θ for quality derives the following utility with firm i:

$$U_i(\theta) = \begin{cases} \theta s_i - p_i & \text{if she buys from firm } i \\ 0 & \text{otherwise} \end{cases}$$

Model of vertical differentiation

Other assumptions:

- $s_2 > s_1$: firm 2 is the high-quality firm, firm 1 the low-quality firm
- We define $\Delta s = s_2 - s_1$ as the difference in quality
- Enough heterogeneity between consumers:

$$\bar{\theta} \geq 2\underline{\theta}$$

  (otherwise the low-quality firm is excluded)
- The market is "covered" in equilibrium (i.e., all consumers buy a good):

$$s_1\underline{\theta} \geq p_1 = c + \frac{\bar{\theta} - 2\underline{\theta}}{3}(s_2 - s_1)$$

Roadmap for solving the model

This model is solved in a similar way as the Hotelling model

We begin by solving for the **price competition equilibrium** in the second stage

1. We first determine the marginal consumer
2. This gives the demand functions for each firm
3. We then find the best response functions in prices
4. The intersection of the reaction functions gives the equilibrium prices

Next, we determine the **equilibrium quality choices** in the first stage, assuming that firms expect the price equilibrium to prevail in the second stage.

Equilibrium prices

The **equilibrium prices at the second stage** are:

$$p_1^\star = c + \left(\frac{\bar{\theta} - 2\underline{\theta}}{3}\right)\Delta s$$

$$p_2^\star = c + \left(\frac{2\bar{\theta} - \underline{\theta}}{3}\right)\Delta s$$

Vertical differentiation Δs (like horizontal differentiation) gives firms market power: $p_i^\star > c$

The price of the firm with high quality (firm 2) is *higher* than the price of the low quality firm (firm 1): $p_2^\star > p_1^\star$

The price gap is equal to $p_2^\star - p_1^\star = \frac{\Delta s}{3} \rightarrow$ increases in degree of differentiation between firms

Quality choices

Assume that $s \in [\underline{s}, \bar{s}]$. Why would someone want to choose the low quality?

What are the Nash equilibria?

Equilibrium of the game of vertical differentiation

There are two Nash equilibria, such that one firm offers the lowest quality and the other offers the highest quality.

$\rightarrow$ Same principle of maximum differentiation as in the Hotelling model

If the game is played **sequentially**, the firm that plays first chooses the high quality.

Take-aways

- Firms try to distinguish themselves from their competitors by developing differentiation strategies that allow them to earn higher profits.
- In the Hotelling model with quadratic transportation costs and exogenous locations, firms charge a price equal to marginal cost plus transportation costs: $p^* = c + t$
- In the Hotelling model with quadratic transportation costs, when firms choose their locations, they choose to differentiate as much as possible. Differentiation is excessive compared to the social optimum.
- In the Salop model, the long-run equilibrium price increases with the entry costs.
- There is too much entry compared to the social optimum (brand proliferation strategy).
- Vertical differentiation (e.g., differentiation in quality) also allows firms to charge higher prices.

Course materials are based on the Industrial Organization course by Michele Fabi and Marc Bourreau, whom I gratefully acknowledge for generously sharing their materials.